



The influence of a non-uniform heat source/sink and Joule heating on the convective motion of a micropolar fluid in a chemically radiative MHD medium across a stretched sheet

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Abstract. The objective of the present exploration is to examine impactions of radiation, a non-uniform intensity source, and a permeable medium on a temperamental MHD blended convective micropolar liquid over an extended sheet subject to Joule heating. To transform the formulated problem into ordinary differential equations, the applicable similarity transformation is implemented. By utilizing R-K-F 4th-5th order approach with shooting method with MATLAB, the numerical solution is obtained. For the relevant profiles, the dimensionless parameters are visually displayed and described. Skin friction, the Nusselt number, and the Sherwood number have all been calculated using the answer found for the velocity, temperature, and concentration. With the assistance of line graphs, the impact of different flow factors being introduced into the problem is addressed. This research is conducted on the implications of MHD, porous, thermal radiation, viscous dissipation, Joule heating, non-linear thermal radiation and chemical reaction. For large values of micropolar parameter, the temperature is reduced and velocity and angular momentum distributions are raised. With the thermal radiation parameter, the temperature distribution gets better and thermal boundary layer is improved while the large values of Eckert number and non-uniform heat source or sink parameters, thermal boundary layer is improved. The higher thermal conductivity is proportional to the thickness of the thermal boundary layer. The concentration profile degrades with higher Schmidt number and chemical reaction parameter values. The current examination pertains to the significant subject matter of cooling of systems, artificial heart identification, oil-pipelined frictions, flow-tracers.

Keywords: radiation, Joule heating, injection/suction, heat sink/heat source, functional materials

Funding. The work was funded by the Ministry of Science and Higher Education of the Russian Federation (Project no. FZZS-2024-0003).

For citation: Dharmiah G., Sidorov D.N., Noeiaghdam S., Panov V.P. The influence of a non-uniform heat source/sink and Joule heating on the convective motion of a micropolar fluid in a chemically radiative MHD medium across a stretched sheet. *iPolytech Journal*. 2024;28(3):435-452. <https://doi.org/10.21285/1814-3520-2024-3-435-452>. EDN: ZFGRZC.

ЭНЕРГЕТИКА

Научная статья

УДК 54-79

Влияние неравномерного источника/стока тепла и Джоулева нагрева на конвективное движение микрополярной жидкости в химически излучающей МГД-среде поперек растянутого листа

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Резюме. Проведенное исследование направлено на изучение динамики воздействия излучения, источника неоднородной интенсивности и проницаемой среды на магнитную гидродинамику смешанной конвек-

тивной микрополярной жидкости на протяженном слое плоской пластины, подверженной джоулевому нагреву. Исходная задача сведена к решению обыкновенных дифференциальных уравнений. Для построения численного решения обыкновенных дифференциальных уравнений использованы методы Рунге–Кутты–Фельберга четвертого и пятого порядков и метод стрельбы. В качестве программной среды и для визуализации численных результатов моделирования использован пакет MATLAB. Для соответствующих профилей потоков найдены безразмерные параметры. Исследовано поведение поверхностного натяжения. Число Нуссельта и число Шервуда рассчитаны с учетом скорости потока, температуры и концентрации. Динамика влияния различных параметров потока представлена в графическом виде. Моделирование проведено с учетом магнитогидродинамики потока, пористой структуры конвективной микрополярной жидкости на протяженном слое плоской пластины, влияния вязкой диссипации, нагрева Джоуля, а также нелинейного теплового излучения. Установлено, что для больших значений микрополярного параметра температура снижается, а скорости потока и углового импульса увеличиваются. С использованием параметров теплового излучения с учетом больших значений Эккерта и неравномерного источника тепла распределение температуры и теплового пограничного слоя улучшается. Более высокая теплопроводность пропорциональна толщине теплового пограничного слоя. Профиль концентрации ухудшается с ростом числа Шмидта. Проведенное численное исследование магнитной гидродинамики смешанной конвективной микрополярной жидкости на протяженном слое плоской пластины касается важных прикладных задач охлаждения в системах, таких как анализ трения в масляных трубопроводах, исследованиях сердечно-сосудистой системы, а также может использоваться для построения систем мониторинга потоков.

Ключевые слова: излучение, Джоулев нагрев, впрыск/всасывание, теплоотвод/источник тепла, функциональные материалы

Финансирование. Работа выполнена в рамках государственного задания Министерства науки и высшего образования Российской Федерации (проект FZZS-2024-0003).

Для цитирования: Дхармая Г., Сидоров Д.Н., Нойягдам С., Панов В.П. Влияние неравномерного источника/стока тепла и Джоулева нагрева на конвективное движение микрополярной жидкости в химически излучающей МГД-среде поперек растянутого листа // iPolytech Journal. 2024. Т. 28. № 3. (In Eng.). С. 435–452. <https://doi.org/10.21285/1814-3520-2024-3-435-452>. EDN: ZFGRZC.

INTRODUCTION

In many engineering and industrial processes, the flow of a boundary layer composed of an incompressible liquid across a stretched sheet is a regular occurrence. Over the last several decades, there has been an influx of researchers into this topic. A stretchable surface is an area that is supported at one end and moves in reaction to a tug at that end. This kind of surface may be pulled or stretched in either direction. During the process of manufacturing sheets, molten material is removed from the slit and then stretched to the necessary dimensions. Because of this, the qualities of a product manufactured by industrial extrusion will depend on the rates at which the sheet is stretched and cooled after being produced. Nevertheless, the investigation of laminar flow over a stretched surface is an integral part of a wide variety of manufacturing procedures, such as the production of glass fiber and paper, metal spinning, the reinforcement of copper wires, and hot rolling. The velocity at which the sheet is dragged away from the emulsion slit may be defined in a number of different ways, including linearly, exponentially, and nonlinearly, among other conceivable ways. Several situations that occur in the real world for the use of a nonlinear stretching sheet. In light of this, it is sometimes needed to assume that the velocity of the sheet changes nonlinearly as a function of distance from the slit. This is due to the fact that the slit causes the velocity of the sheet to change in a nonlinear fashion. Within the scope of this work, our primary focus is on nonlinear stretching sheets. As a consequence of this, external flow control devices like a magnetic field are necessary in order to guarantee that the proper feature is shown.

Theory of fluids with microscopic rotation, first investigated by Eringen [1], can provide an explanation for the flow behavior. Unlike the classical Newtonian fluids theory, which does not take into account micro- and nano-scale movements of fluid components on a microscopic scale, the theory of micro rotation fluids can. Stiff micromolecules are suspended in a diluted solution in these liquids; Spin-inertia controls their independent movements, which maintain tension and body moments. Eringen's original microcontinuum theory is comprised of the micropolar, microstretch, and micromorphic theories [2–5]. Micropolar fluids refer to non-Newtonian fluid models that provide a means of studying the properties and behavior of unique lubricants, colloidal suspensions, polymeric fluids, and liquid crystals. The effects of heat source and sink on heat transmission are also important to take into account in the context of numerous physical issues. The heat distribution in the fluid, which may be generated or absorbed by the system,

may effects over particle depositions like nuclear reactors, electronics, and semiconductors. It's feasible to arrange heat sources and sinks as one or the other steady, area or temperature-subordinate, or both. In this article, we'll discuss heat sources and sinks that fluctuate regarding both area and temperature. Within the sight of warm radiation/substance response, Mabood et al. [6, 7] looked into heat source affected magneto non-Darcian convected micropolar flow on a stretched plane. Numerous researchers have investigated the significance of heat sink/source in micropolar liquids with various factors [8–12].

The importance of understanding radiation's effect on MHD flows and heat transfers from an industrial perspective has grown. Many cycles in the designing process happen at high temperatures, making knowledge of radiation heat motion crucial for constructing sturdy hardware, thermal energy stations, gas turbines, and a broad variety of propulsion equipment for aircraft, rockets, satellites, and spacecraft. Using exact analytical techniques, Rashidi [13] examined the impact of heat radiation on a micropolar fluid trapped in a porous medium. Numerous industrial processes, such as hot rolling, chemical plating of flat plates, and polymer extrusion [14], make use of chemical reactions. Late writing on the effect of substance responses in different stream fields is given in Refs. [15–19]. Since they were on a similar request thus little, the impacts of gooey and Joule warming were dismissed [20]. Heat transport is significantly influenced by viscous dissipation. While managing extremely thick liquids, the dispersal term becomes huge. While working at high temperatures, as in polymer creation, thick dissemination can't be disregarded. A few authors [21, 22] review the synergistic effect of Joule warming and thick dissemination on various kinds of streams [23–27]. Contain additional MHD studies. Morozova et al discussed propagation of heat and angular velocity for micropolar medium in [28]. There are lots of ways we can use nanomechanical modeling to understand and study very tiny structures. We can also use theories of bending and twisting in various elastic materials including functional materials. You can find examples of this in some books or papers [29–31].

There are several areas where knowledge gained from studying two-layer limit layer stream, intensity, and mass exchange across a nonlinear extending surface might be useful. Streamlining the removal of plastic sheets, building up cycles of metal plates in cooling showers, and removing polymer sheets from colors are only some of the more recent applications of thick stream across an extended sheet. When making these sheets, the dissolve that results from a cutting process is then prolonged till the desired thickness is reached. The eventual outcomes of wanted qualities are prominently affected by the extending rate, the pace of the cooling simultaneously, and the method involved with extending. Vajravelu [32] concentrated on the stream and intensity move qualities in a thick liquid over a nonlinearly extending sheet without heat dispersal impact. Cortell [33–34] has dealt with thick stream and intensity move over a nonlinearly extending plane. Raptis and Perdakis [35] concentrated on thick stream over a nonlinear extending sheet within the sight of a substance response and attractive field. Radiation impact with magneto stream owing to a spreading regime in a permeable media were discussed by Abbas and Hayat [36]. Studying the relevance of similarity response for stream and intensity move of a silent liquid across a nonlinear expanding surface, Cortell [37] found some interesting results. Solutions for a stream across a nonlinearly expanding sheet with synthetic response and attractive field was found by Awang and Hashim [38].

This article inspects the impacts of a synthetic response and a permeable media on the MHD blended convection stream of a micropolar liquid across an extended sheet, as well as the overseeing conditions for this issue. In this exploration, the limit layer is represented by a bunch of conditions including the coherence, force, miniature turn, energy, and focus conditions. To tackle limit layer issues, an arrangement of incomplete differential conditions (PDEs) should be changed into a Tribute framework by means of closeness change. Runge-Kutta-Fehlberg fourth-fifth request approach is then used to take care of the issue mathematically.

PRELIMINARIES

Here, we presented some main definitions and preliminaries [11–14]. Liquid mechanics is among the finest seasoned parts of material science which manages the way of behaving

of liquids (fluids and gases) very still as well as moving. Like different parts of science, liquid mechanics can be extensively named liquid insights, liquid kinematics and liquid elements. Liquid elements manages the investigation of movement of liquids, the powers that are answerable for this movement and the collaboration of liquids with solids. The hypothetical investigation of movement of liquids is the best, captivating and helpful utilization of science. Liquid elements is a profoundly evolved part of science which has been the constant and growing exploration movement beginning around 1840. It has its own significance in the fields of designing, physiology, satellite innovation, and so forth. Liquid elements in without a doubt contacts pretty much every part of our life. It is examined both hypothetically and tentatively and the outcomes are portrayed both mathematically and truly. The journey for more profound comprehension of the subject has assisted the improvement of the subject as well as recommended the advancement in supporting regions with enjoying applied arithmetic, mathematical registering and exploratory procedures.

Liquids are not sensibly compressible except under the action of heavy forces. Hence, for all practical considerations they are taken as incompressible. Gases, on the other hand, are widely compressible whenever external pressures are exerted. Because of this behavior, gases are considered as compressible fluids in general.

In liquids, the erosion between the particles is as digressive or shearing powers (stresses). The inside rubbing among the atoms of liquid which offers protection from the deformity of the liquid is called thickness. In examining fluid behavior and smooth movements near a limit, this property is crucial. The proportion of thickness is known as the coefficient of consistency. It is an actual property. In spite of the fact that gases as well as fluids have consistency, the thickness of gases is less clear in regular daily existence.

LAMINAR FLOW

A stream where every liquid molecule has an unequivocal way and the way of one molecule doesn't cross the way of some other molecule is supposed to be laminar stream. This stream happens when an exceptionally gooey liquid like sloppy water, streams at an extremely low speed through a pipeline. Implies, it happens at a low speed so that powers because of the consistency are transcendent over the inner powers. The thickness of the liquid prompts relative movement with the liquid as the liquid layer slides over one another, which thusly leads to shearing stresses. Assuming the Reynolds number is under 2000, the stream is called laminar stream.

TURBULENT FLOW

A stream wherein liquid particles don't have an unmistakable way and the way of one molecule crosses the way of different particles during stream is called non-laminar or tempestuous stream. This stream happens when a liquid of low thickness like petroleum, courses through a pipeline.

The event of fierce stream is more regular than that of laminar stream. The speed circulation in tempestuous stream is somewhat uniform. On the off chance that the Reynolds number is more than 4000, the stream is called violent. Assuming the Reynolds number lies somewhere in the range of 2000 and 4000, the stream might be tempestuous or laminar.

STEADY FLOW

A stream where the properties and conditions related with the movement of the liquid are free of time so the stream design stays unaltered with time is supposed to be a consistent stream.

For a steady flow $\frac{\partial}{\partial t}(\zeta) = 0$ when the fluid is at any, where ζ is a parameter.

UNSTEADY FLOW

A stream where the properties and conditions related with the movement of the liquid are subject to time so the stream design fluctuates with time is supposed to be a flimsy stream. Insecure stream is found when a compressible liquid (i.e., gas) courses through a pipeline.

For an unsteady flow $\frac{\partial}{\partial t}(\zeta) \neq 0$ when the fluid is at any, where ζ is a parameter.

HEAT TRANSFER

The concept of heat transmission is crucial to many fields of study and development. It is a fundamental part of thermal engineering. Heat transfer happens whenever there is a temperature differential inside or between two media.

The uses of intensity move are different both in nature and businesses like warming, ventilating and cooling frameworks, nuclear energy stations, fridges and intensity siphons, gas detachment and liquefaction, cooling machines, warming up or chilling off of creation parts, sunlight based warm frameworks, and so on. The study of intensity move is likewise expected in thermodynamics, liquid mechanics, material science, mechine plan, and so forth. The various kinds of intensity move are typically alluded to as methods of intensity move. There are three essential systems during the time spent heat move as per which intensity can move from a high-temperature locale to a low-temperature district. All intensity move processes include at least one of these modes. The three methods of intensity move are:

- conduction;
- convection;
- radiation.

CONDUCTION

The transfer of thermal energy between particles may be conceptualized as a transfer of intrinsic energy. Whether the body is at rest or in motion, heat will always flow kinematically or by the direct influence of particles from a region of high temperature to a region of low temperature. The energy moved by conduction and the intensity move rate per unit region is corresponding to the typical temperature angle, i.e,

$$\frac{q}{A} \approx \frac{\partial T}{\partial x}.$$

At the point when the proportionality steady is presented, we have

$$q = -K A \frac{\partial T}{\partial x},$$

where q is the heat transfer rate, A is the surface area through which heat flows and $\frac{\partial T}{\partial x}$ is the temperature gradient in the direction of heat flow. The second rule of thermodynamics states that heat must travel from hotter to cooler regions, hence the positive constant K denoting the thermal conductivity of a material must be prefixed with a negative sign.

CONVECTION

Convection is the exchange of intensity energy because of the movement of warmed liquid particles. This motion occurs into a fluid or within a fluid. It cannot occur in solids due to the atoms not being able to flow freely.

We have three types of heat convections as:

- free or natural;
- forced;
- mixed.

MASS TRANSFER

Changes in the relative abundance of different species in a given environment might result in the transfer of mass. The concentration gradient aids in the speed of the mass transfer. The direction of a reacting concentration gradient is always the direction of mass transfer. Mass transfer is a common occurrence in stellar structure theories. It may be used in a broad variety of technological, engineering, and scientific applications. Common mass transfer processes may be found in the fields of electronics, architecture, aviation, metallurgy, environmental engineering, refrigeration, and manufacturing. Geophysics, astronomy, meteorology, and even the preparation of food all need an understanding of heat and mass transfer. The foundation of many biological and chemical processes is mass transfer. Oxygenation of blood, renal-functioning, osmosis,

medicine and food absorption, fog dispersion, and other biological processes are only a few examples. Chemical processes including distillation, gas absorption, solids and liquids interaction from their combination, etc. all use mass transfer applications. Mathematical analysis benefits greatly from the parallels between heat and mass transmission.

Due to the following two methods, mass transfer appears:

- (i) Diffusion mass transfer
- (ii) Convective mass transfer

(i) Mass diffusion:

Diffusion is the movement of molecules, species, or particles from one component to another to effect a change in mass. Diffusion mass transfer may be brought on by either a concentration gradient or a temperature gradient or even a pressure gradient.

(ii) Mass convection:

The dynamic property of the moving fluid frequently helps in transfer of mass from a flowing fluid to a stationary surface or between two immiscible moving fluids that are at different speeds on a moveable interface. This kind of transmission is known as convective mass transfer. The dynamic capabilities and transmission properties of the fluid are crucial. The process of convective heat transmission is identical to that of mass transfer. Convective mass transfer may be split into two types: natural or free convective mass transfer, and forced convective mass transfer.

Natural convective mass transfer occurs when species move due to density variations caused by temperature or concentration differences or mixes of varying composition, whereas forced convection mass transfer depends on external causes.

Mathematical formulation:

Over a permeable stretched sheet, we study with Micropolar magnetohydrodynamic fluid in an unstable viscous incompressible 2D mixed convection flow.

1. The fluid is in a constant condition at time $t < 0$, whereas the heat and mass fluxes are in an erratic state at time $t \geq 0$.
2. The y-axis is measured perpendicular to the s-axis, along the lengthening/shortening sheet.
3. The magnetic field strength B is applied along the y-axis (see Fig. 1).
4. Chemical processes, heat dissipation through viscous dissipation in a porous medium, and thermal radiation are all accounted for.

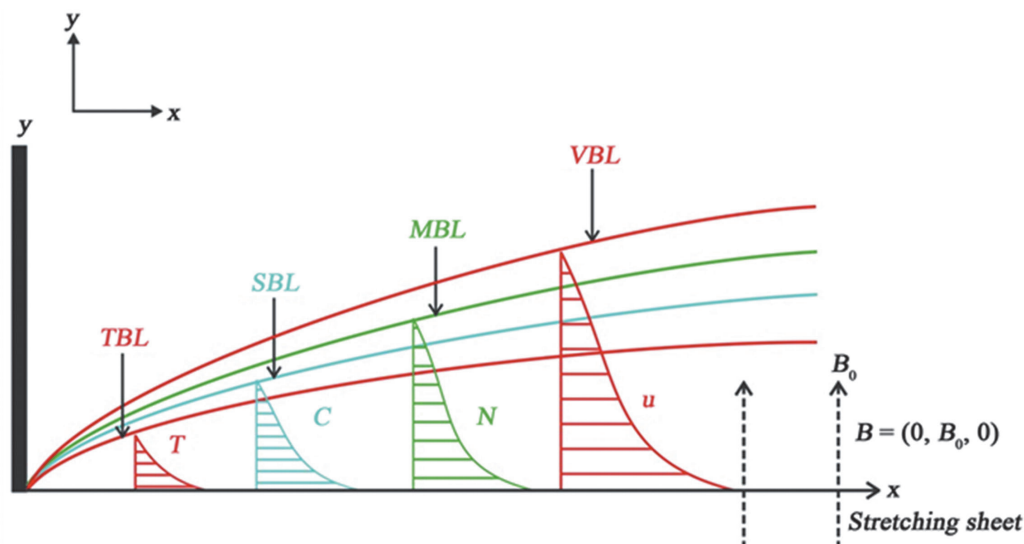


Fig. 1. Physical model of the flow

Рис. 1. Физическая модель потока

The governing equations of the flow under the aforementioned presumptions are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\frac{\mu + \kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \left(\frac{\kappa}{\rho} \right) \frac{\partial w}{\partial y} - \frac{\sigma B_0^2}{\rho} u - \frac{v}{\rho K_p^*} u; \quad (2)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \frac{\gamma}{\rho j} \frac{\partial^2 w}{\partial y^2} - \frac{\kappa}{\rho j} \left(2w + \frac{\partial u}{\partial y} \right); \quad (3)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \left(\frac{\mu + \kappa}{\rho c_p} \right) \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B^2}{\rho c_p} u^2 + q'''; \quad (4)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - kr(C - C_\infty). \quad (5)$$

Boundary conditions:

$$u = \lambda U_w(x, t), \quad v = v_w, \quad w = -\frac{1}{2} \frac{\partial u}{\partial y}, \quad T = T_w(x, t), \quad C = C_w(x, t) \quad \text{at } y = 0,$$

$$u \rightarrow 0, \quad w \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad \text{as } y \rightarrow \infty. \quad (6)$$

The heat source or sink is not consistent, thus we obtain

$$q''' = HU_w(x, t)k / xv, \quad (7)$$

where $H = (A^*(T_w - T_\infty)f' + B^*(T - T_\infty))$; the A^* and B^* characteristics of the interior heat source/sink space change with temperature.

A positive value for A^* and a negative value for B^* indicate a heat source and a heat sink, respectively.

Rosseland diffusion approximation is used to model thermal radiation and thereby calculate the radiative heat flow.

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}, \quad (8)$$

where k^* refers Rosseland coefficient and σ^* refers constant of Stefan-Boltzman.

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \quad (9)$$

Invoking equations (8) and (9), equation (4) becomes

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\frac{\kappa}{\rho c_p} + \frac{16T_\infty^3}{3\rho c_p k^*} \right) \frac{\partial^2 T}{\partial y^2} + \left(\frac{\mu + \kappa}{\rho c_p} \right) \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B^2}{\rho c_p} u^2 + q'''. \quad (10)$$

The following non-dimensional variables are included in order to find a similarity solution to the issue:

$$\begin{aligned} \zeta &= \sqrt{\frac{a}{v(1-et)}} y; & \psi &= \sqrt{\frac{va}{(1-et)}} x f(\zeta); & w &= \sqrt{\frac{a^3}{v(1-et)^3}} x h(\zeta); \\ \theta &= \frac{T - T_\infty}{T_w - T_\infty}; & \phi &= \frac{C - C_\infty}{C_w - C_\infty}; & B &= \frac{B_0}{\sqrt{1-et}}; & kr &= \frac{k_0}{(1-et)}, \end{aligned} \quad (11)$$

where $\psi(x, y)$ is the stream function specified by and ζ is the similarity variable.

$$u = \frac{\partial \psi}{\partial y} = U_w f'(\zeta); \quad v = -\frac{\partial \psi}{\partial x} = -\sqrt{\frac{va}{(1-et)}} f(\zeta),$$

which identically satisfies equation (1). The following set of ordinary differential equations is created using similarity variables (11), equations (2), (3), (9) and (5):

$$(1 + K) f''' + ff'' - f'^2 - \tau \left(f' + \frac{1}{2} \zeta f'' \right) + Kh' - (M + Kp) f' = 0; \quad (12)$$

$$\left(1 + \frac{K}{2}\right)h'' + \left|\frac{f}{h} \frac{f'}{h'}\right| - \tau\left(\frac{3}{2}h + \frac{1}{2}\zeta h'\right) - K(2h + f'') = 0; \quad (13)$$

$$\left(1 + \frac{4}{3}Nr\right)\theta'' + \text{Pr}\left|\frac{f}{\theta} \frac{f'}{\theta'}\right| - \text{Pr}\tau\left(\theta + \frac{1}{2}\zeta\theta'\right) + \text{Pr}Ec(1 + K)f''^2 + \text{Pr}MEcf'^2 + A^*f' + B^*\theta = 0; \quad (14)$$

$$\phi'' + Sc\left|\frac{f}{\phi} \frac{f'}{\phi'}\right| - Sc\tau\left(\phi + \frac{1}{2}\zeta\phi'\right) - Sc\gamma\phi = 0. \quad (15)$$

And we have the following boundary conditions:

$$f(0) = S, \quad f'(0) = \lambda, \quad h(0) = -\frac{1}{2}f''(0), \quad \theta(0) = 1, \quad \phi(0) = 1, \quad f'(\infty) = 0, \quad h(\infty) = 0, \\ \theta(\infty) = 0, \quad \phi(\infty) = 0, \quad (16)$$

where the parameters are specified in the following form which the notation primes denotes differentiation with respect to ζ

$$\tau = \frac{a}{e}; \quad K = \frac{\kappa}{\mu}; \quad M = \frac{\sigma B_0^2}{\rho a}; \quad K_p = \frac{\nu}{\rho K_p^* a}; \quad Gr = \frac{g\beta(T_w - T_\infty)x^2}{aU_w^3}; \quad Gc = \frac{g\beta^*(C_w - C_\infty)x^2}{\nu U_w}; \\ S = -\frac{\nu_w}{\sqrt{av}}; \quad \text{Pr} = \frac{\nu}{\alpha}; \quad Ec = \frac{U_w^2}{c_p(T_w - T_\infty)}; \quad R = \frac{4\sigma T_\infty^3}{\kappa k^*}; \quad Sc = \frac{\nu}{D_m}; \quad \gamma = \frac{k_0}{a}.$$

The local heat and mass transmission rates as well as the pair stress and skin friction factors, which are:

$$C_{fx} = \frac{\tau_w|_{y=0}}{\rho U_w^2} = 2(1 + K)\text{Re}_x^{\frac{1}{2}}f''(0); \quad (17)$$

$$C_{sx} = \frac{\gamma a U_w}{\nu(1 - et)}h'(0); \quad (18)$$

$$Nu_x = -x(T_w - T_\infty)^{-1}\frac{\partial T}{\partial y}\bigg|_{y=0} = -(\text{Re}_x)^{\frac{1}{2}}\theta'(0); \quad (19)$$

$$Sh_x = -x(C_w - C_\infty)\frac{\partial C}{\partial y}\bigg|_{y=0} = -(\text{Re}_x)^{\frac{1}{2}}\phi'(0), \quad (20)$$

where $\text{Re}_x = \frac{U_w x}{\nu}$ is the Reynolds number.

NUMERICAL ANALYSIS

The fourth-fifth-order Runge-Kutta technique is one of the most well-known constant-step procedures. The Runge-Kutta approach, with some reasonable approximation adjustments, may achieve the precision of a Taylor Series approximation without resorting to more complex derivative computations. This technique could be seen as the ancestor of many other techniques. On the other hand, the Runge-Kutta-Fehlberg technique, which uses an adjustable step size, provides more accurate error estimates than the Runge-Kutta method, which uses a fixed step size. At each stage, the Runge-Kutta-Fehlberg approach detailed the computation using two Runge-Kutta procedures of different order (RK4 and RK5). If the differences between the two are small enough, the following stage with the same step size as before. The step size should be decreased to maintain the same level of precision. The step size is raised if the answers match to more digits of precision than are needed. Therefore, we infer that the step size is automatically organized as a recombination to the computation truncation errors in the one-step algorithm technique coupled with an adaptable step size. Nonlinear models are no match for this technique, which has been proved to function in a broad variety of other contexts, including deterministic and stochastic, linear and nonlinear, difficulties in physics, biology, and chemical processes, and so on.

A set of regular nonlinear DEs (12-15) with boundary conditions (16) is built using the Runge-Kutta Felherberg-45 process with shooting approach.

Follow the steps below to figure out how to fix the problem:

- i. The BVPs undergo a transmute to become the IVPs.
- ii. Assume suitable significant digits for η_{∞} .
- iii. Following the selection of baseline prediction values, at random, Secant method, utilized to provide further approximative results for those assessments.
- iv. Use the aforementioned approximations to compute IVP using RKF-45 approach with step variance $h = 0.001$.
- v. Iterations of the RKF-45 algorithm were performed until 10^{-6} order of convergence results was achieved.
- vi. The correct step length, represented by “h”, is determined by an iterative process in this approach.
- vii. At each stage, we generate and compare two new estimates.
- viii. The findings are used to fine-tune the step size until the results of the most recent two estimates are consistent with one another.

Applying the RK of order 4 method yields the required result for the given problem.

$$y_{m+1} = y_m + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4104}k_4 - \frac{1}{5}k_5.$$

An improved solution may be obtained by using the R-K method of order 5:

$$y_{m+1} = y_m + \frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{25}k_6.$$

The following six values are needed for each step:

$$k_1 = hf(y_k, t_k);$$

$$k_2 = hf\left(y_k + \frac{h}{4}, t_k + \frac{h}{4}, k_1\right);$$

$$k_4 = hf\left(y_k + \frac{12h}{13}, t_k + \frac{1932}{2197}k_1 - \frac{7200}{2197}\right);$$

$$k_5 = hf\left(y_k + h, t_k + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4\right);$$

$$k_6 = hf\left(y_k + \frac{h}{2}, t_k - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 - \frac{1859}{4104}k_4 - \frac{11}{40}k_5\right)$$

This is achieved by first inserting additional variables into the coupled ODEs and converting them to first order ODEs. The R-K-F 4th-5th order technique is utilized numerically solve the system of transformed ordinary differential equations. The effects of parameters on the velocity $f'(\zeta)$ micro-rotation $h(\zeta)$, temperature $\theta(\zeta)$ and concentration $\phi(\zeta)$ profiles are analyzed. Also, we found the results based on $f''(0)$, $h'(0)$, local Nusselt number $\theta'(0)$ and local Sherwood number $\phi'(0)$. Comparison of results are shown in Table.

The Nusselt number is compared with its present results at $Pr=0.733$, $K=0.2$, $R=\delta=A^*=B^*=Kp=M=\gamma=0.0$

Число Нуссельта сравнивается с его текущими результатами при $Pr=0,733$, $K=0,2$, $R=\delta=A^*=B^*=Kp=M=\gamma=0,0$

S	Ref [39]	Current		Ref [39]	Current		Ref [39]	Current
3.0	-0.2492	-0.2493	1	-0.2492	-0.249253	0	0.8889	0.888959
3.5	0.1291	0.12916	2	0.1466	0.146611	1	0.5444	0.54445
4.0	0.4599	0.45992	3	0.4659	0.465979	2	0.1709	0.170959
4.5	0.7506	0.75069	4	0.7155	0.715571	3	0.2492	-0.249249
5.0	1.0127	1.0128	5	0.9165	0.916569	4	-0.7330	-0.733087

RESULTS AND DISCUSSIONS

The findings demonstrate the impact of the non-dimensional governing factors on the flow's velocity, microrotation, temperature, and concentration distributions, providing a clear understanding of the issue. Fig. 2–9 illustrate how the magnetic field and the permeability of the porous media affect the distributions of velocity, microrotation, temperature, and concentration in both the suction and injection cases. We infer from Fig. 2–5 that raising the magnetic parameter and the permeability of the porous media lower the velocity profile and increase the microrotation close to the boundary, and that the microrotation reverses its effect at $\zeta_{\infty}=2.0$ level, causing a drop in angular momentum. Owing to the drag force that is created when a magnetic parameter is given to an electrically conducting fluid, which causes a drop in velocity. Increased values of magnetic factor on angular momentum suggest an increasing trend, which is caused by the higher influence of stimulated Lorentz force in the non-negative x-orientation. When there is a greater disparity in speeds, the thickness of the boundary layer that corresponds to those speeds will decrease. Because of this overextension in the distribution of velocities, we can see that the velocity of the fluid surrounding the stretched sheet is lower than its velocity in the free stream. A higher value for the modified magnetic quantity is correlated to a lower value for the outside electrical profile, which in turn causes a higher value for the velocity field. In addition to this, as the boundary layer grew, there was a downward trend in the magnitude of the wall velocity gradient.

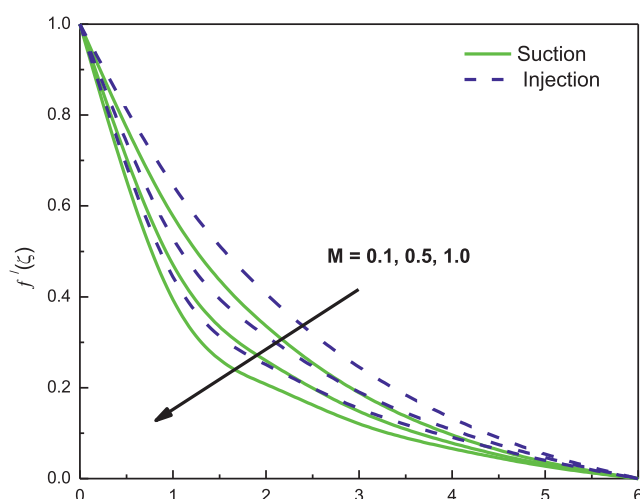


Fig. 2. The effect of M on the velocity profile

Рис. 2. Влияние величины M на профиль скорости

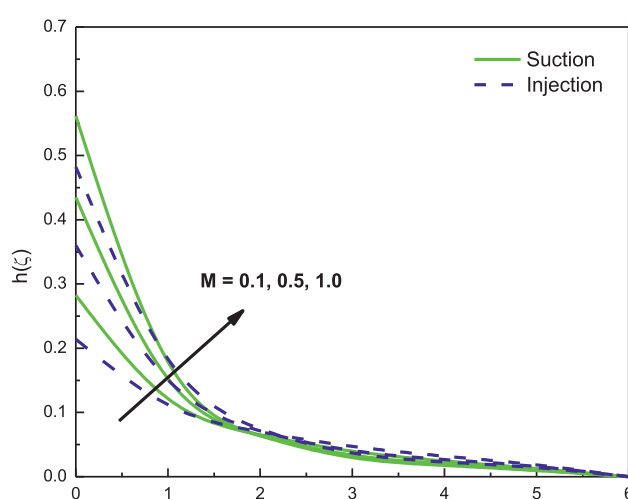


Fig. 3. The effect of M on the angular momentum profile

Рис. 3. Влияние величины M на профиль углового момента

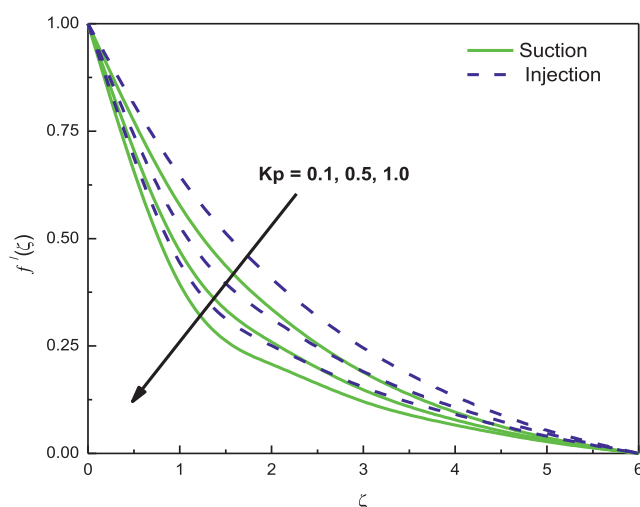


Fig. 4. The effect of K_p on the velocity profile

Рис. 4. Влияние K_p на профиль скорости

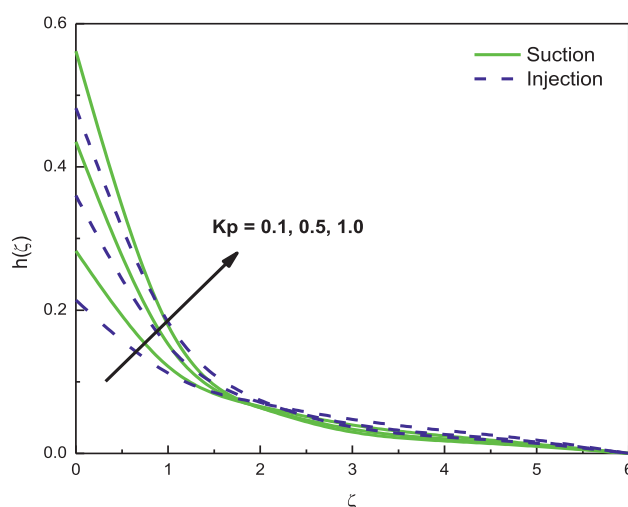


Fig. 5. The effect of K_p on the angular momentum profile

Рис. 5. Влияние K_p на профиль углового момента

Fig. 6–7 show velocity, angular momentum, and temperature distributions for the suction and injection instances, respectively, illustrating the impact of the micropolar parameter. These graphs demonstrate the relationship between the micropolar parameter and changes in velocity, angular momentum, and temperature.

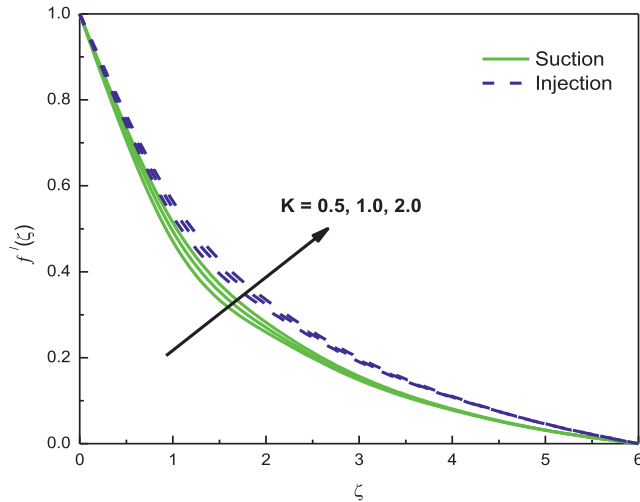


Fig. 6. The effect of K on the velocity profile
Рис. 6. Влияние K на профиль скорости

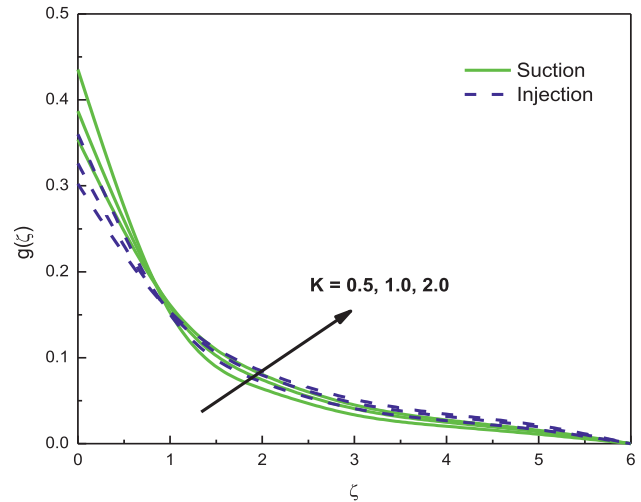


Fig. 7. The effect of K on the angular momentum profile
Рис. 7. Влияние K на профиль углового момента

In general, the angular velocity of an additional material increases as the fluid's consistency weakens. Temperature distribution during suction and injection is shown to be affected by the thermal radiation parameter in Fig. 8.

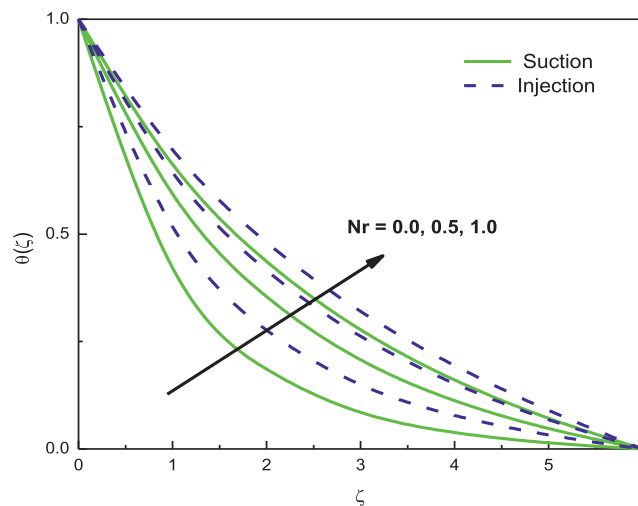


Fig. 8. The effect of Nr on the temperature profile
Рис. 8. Влияние Nr на температурный профиль

These plots show that an increase in the thermal radiation parameter results in a more optimal temperature distribution under both conditions. Both the suction and injection cases may be understood by reference to Fig. 9 and 10, which show how the Eckert number and a non-uniform heat source or sink affect temperature distributions. Statistical analysis reveals that temperature distributions benefit from non-uniform heat sources or sinks and a higher Eckert number. The Eckert number, sometimes known as Ec , does not have any dimensions in continuum mechanics. It does this by describing the connection between the kinetic energy of a flow and the enthalpy difference in the boundary layer. This gives it the ability to characterize heat

transfer dissipation. The Eckert number is a way to quantify the flow's kinetic energy in relation to the enthalpy difference between flows with high viscosity and flows that are dissipative. Fig. 5 illustrates how the Eckert amount, denoted by Ec , behaves in relation to the energy contours. Ec in the flow field increases the energy, which results in a higher fluid temperature in the fluid region due to dissipation caused by viscosity and elastic deformation. This is due to the fact that Ec in the flow field increases the energy. Because of the amount of internal friction heating that occurred between the molecules of the fluid, the quantity of mechanical energy was changed into thermal energy, and this thermal energy is now stored in the fluid. As a consequence of this, an increase in the Eckert number causes an increase in the thermal energy of the flow, which in turn causes an increase in the temperature of the fluid across the thermal boundary layer.

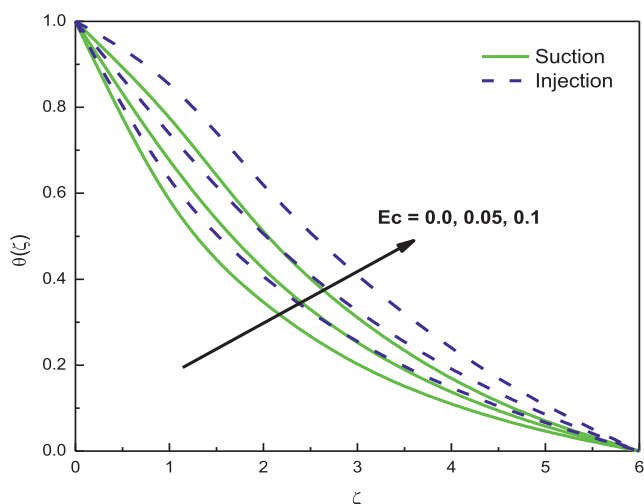


Fig. 9. The effect of Ec on the temperature profile
Рис. 9. Влияние Ec на температурный профиль

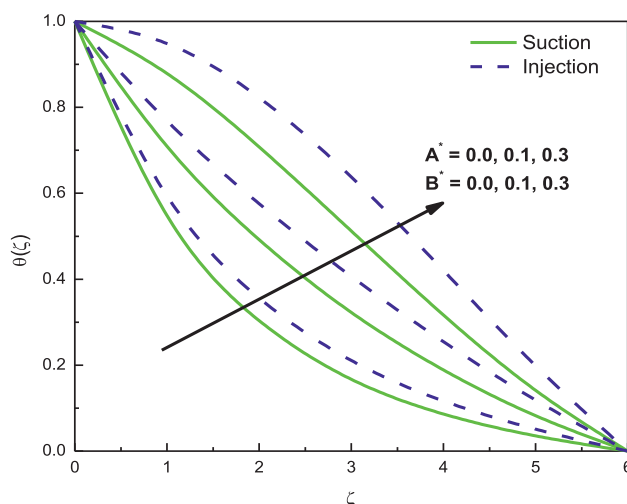


Fig. 10. The effect of points A^* and B^* on the temperature profile
Рис. 10. Влияние точек A^* и B^* на температурный профиль

Additional parameters, such as the unsteady parameter, may impact the distributions of velocities, angular velocities, temperatures, and concentrations in both suction and injection conditions, as illustrated in Fig. 11–14. When the unsteadiness parameter is raised, the graphs show rising trends in velocity, angular momentum, temperature, and concentration.

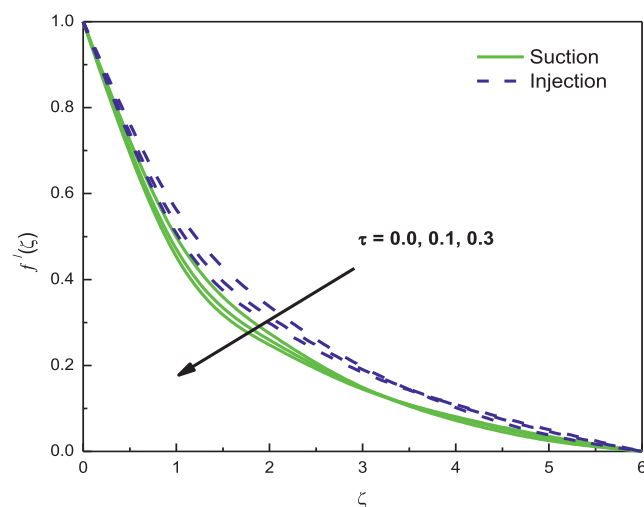


Fig. 11. The effect of τ on the velocity profile
Рис. 11. Влияние на τ профиль скорости

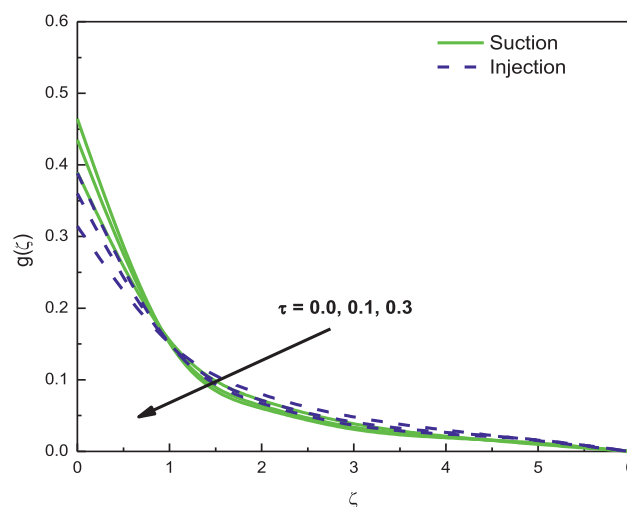


Fig. 12. The effect of τ on the angular momentum profile
Рис. 12. Влияние на τ профиль углового момента

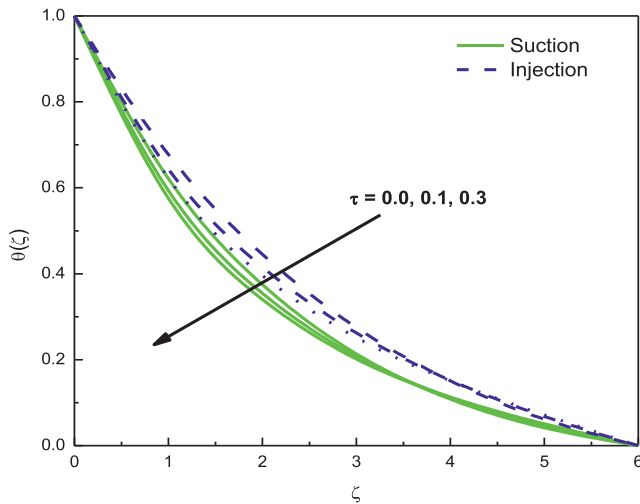


Fig. 13. The effect of τ on the temperature profile
Рис. 13. Влияние на τ температурный профиль

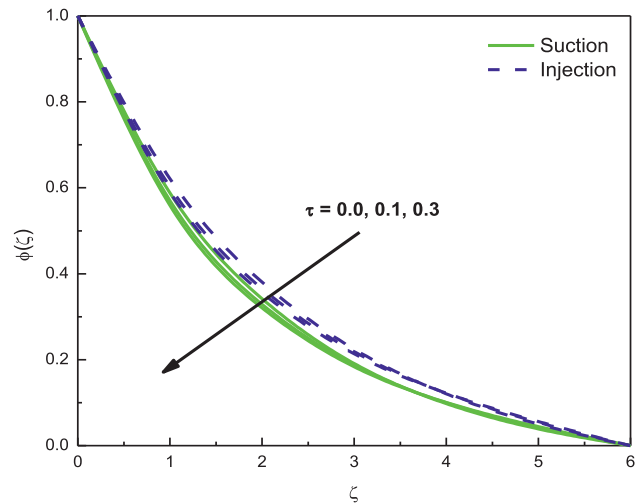


Fig. 14. The effect of τ on the concentration profile
Рис. 14. Влияние на τ профиль концентрации

Fig. 15 and 16 exhibit velocity and concentration curves for suction and injection scenarios, respectively, and illustrate the impacts of the Schmidt number and the chemical reaction parameter. As can be seen in the Fig. 15 and 16, the concentration distributions degrade with an increase in both the Schmidt number and the chemical reaction parameter. The Schmidt number, often known as Sc , is a representation of the connection between momentum and mass diffusivity. It does this by calculating the relative efficiency of momentum and mass transfer in the hydrodynamic (velocity) and concentration (species) boundary layer. The fluid's mass diffusivity is reduced as the value of the Schmidt number increases, which results in a decrease in the concentration profiles. With the typical physical behavior of the parameters of chemical reactions, which indicates that a faster rate of chemical reaction would lead to a thinner concentration boundary layer. It is deduced from the appearance of the reactive species that the solute profiles have been subjected to a destructive chemical reaction. The behavior of the destructive reaction that was caused by the positive values that were investigated in the present scenario may be explained by deriving it from the mathematical expression of the governing equation of the solute profile and examining it.

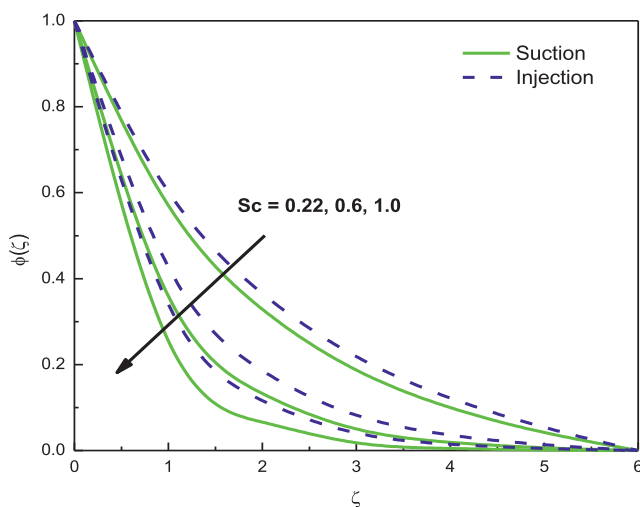


Fig. 15. The effect of Sc on the concentration profile
Рис. 15. Влияние Sc на профиль концентрации

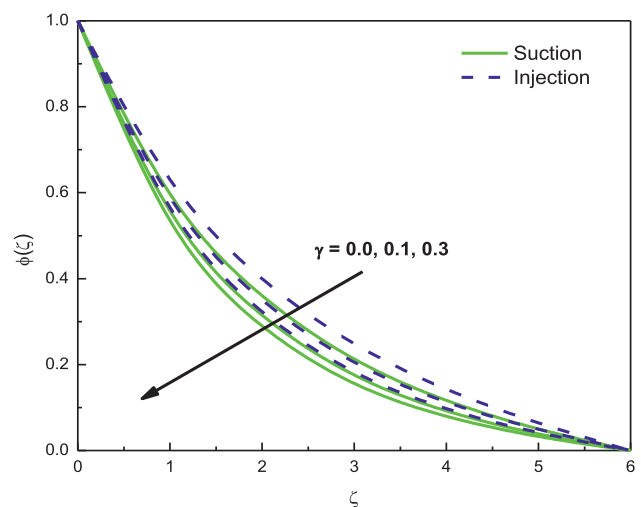


Fig. 16. The effect of γ on the concentration profile
Рис. 16. Влияние на γ профиль концентрации

When γ is increased, it can be seen that there is a corresponding drop in the concentration profile. This is due to the fact that chemical processes taking place in this system are responsible for consuming the chemical, which in turn results in a drop in the concentration profile. The tendency of the first-order chemical reaction to lessen the overshoot in the concentration profiles of the boundary layer is the most notable repercussion of the reaction's first-order status.

CONCLUSIONS

The primary conclusions of the present research are:

1. A rise in the micropolar parameter causes a decline in temperature, but an increase in speed and angular momentum.
2. In both scenarios, the temperature distribution gets better when the thermal radiation parameter is increased.
3. An increase in Eckert number and non-uniform heat source or sink characteristics both improve the temperature profiles.
4. The concentration distributions deteriorate with increasing Schmidt number and chemical reaction parameter.
5. Decreases in the thickness of the concentration boundary layer are frequently seen in conjunction with increases in the rate of chemical reaction.

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Conflict of interests

Denis N. Sidorov has been a member of the editorial board of the iPolytech Journal since 2021, but he did not take part in making decision about publishing the article under consideration. The article was reviewed following the Journal's review procedure. The authors did not report any other conflicts of interest.

The final manuscript has been read and approved by all the co-authors.

Information about the article

The article was submitted 14.03.2024; approved after reviewing 25.04.2024; accepted for publication 01.05.2024.

Заявленный вклад авторов

Все авторы сделали эквивалентный вклад в подготовку публикации.

Конфликт интересов

Сидоров Д.Н. является членом редакционной коллегии журнала «iPolytech Journal» с 2021 года, но не имеет отношения к решению опубликовать эту статью. Статья прошла принятую в журнале процедуру рецензирования. Об иных конфликтах авторы не заявляли.

Все авторы прочитали и одобрили окончательный вариант рукописи.

Информация о статье

Статья поступила в редакцию 14.03.2024 г.; одобрена после рецензирования 25.04.2024 г.; принята к публикации 01.05.2024 г.